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ON SEMIDISCRETIZATION METHODS FOR DIFFERENTIAL INCLUSIONS OF FRACTIONAL ORDER

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Abstract. The report provides semidiscretization diagram for semilinear differential inclusions of fractional order.
Keywords: fractional differential inclusion; semilinear differential inclusion; Cauchy problem; approximation; semidiscretization; fixed point; condensing map; measure of noncompactness

Introduction

Theories of differential inclusions and condensing mappings are of great importance in modern mathematics (see [1], [2]). In our work, we present further development of these theories for differential inclusions of fractional order.

For a semilinear fractional order differential inclusion in a separable Banach space E of the form

$$^CD^qx(t) \in Ax(t) + F(t, x(t)), \ t \in [0, T], \tag{1}$$

consider the problem of existence of mild solutions to this inclusion satisfying the following periodic

$$x(0) = x(T) \tag{2}$$

and anti-periodic

$$x(0) = -x(T), \tag{3}$$

boundary value conditions under the following basic assumptions.

The symbol $^CD^qx$ denotes the Caputo fractional derivative of order $q \in (0, 1)$. We suppose that the linear operator A satisfies condition (A)

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(A) $A : D(A) \subseteq E \to E$ is a linear closed (not necessarily bounded) operator generating a C_0–semigroup $\{U(t)\}_{t\geq 0}$ of bounded linear operators in E

We will assume that the multivalued nonlinearity $F : [0,T] \times E \to Kv(E)$ obeys the following conditions:

$(F1)$ for each $x \in E$ the multifunction $F(\cdot, x) : [0,T] \to Kv(E)$ admits a strongly continuous selection;

$(F2)$ for a.e. $t \in [0,T]$ the multimap $F(t,\cdot) : E \to Kv(E)$ is u.s.c.;

$(F3)$ there exists a function $\alpha \in L^{\infty}_{+}([0,T])$ such that

$$\|F(t,x)\|_E \leq \alpha(t)(1 + \|x(t)\|_E) \text{ for a.e. } t \in [0,T],$$

$(F4)$ *the χ-regularity condition:* there exists a function $\mu \in L^{\infty}([0,T])$ such that for each bounded set $\Omega \subset E$ we have:

$$\chi(F(t,\Omega)) \leq \mu(t)\chi(\Omega),$$

for a.e. $t \in [0,T]$, where χ is the Hausdorff MNC in E.

Along with inclusion (1), for a given sequence of positive numbers $\{h_n\}$ converging to zero consider the inclusion

$$D^q x_h(t) \in A_h x_h(t) + F_h(t, x_h(t)), \qquad t \in [0,T], \tag{4}$$

where $h \in H = \overline{\{h_n\}}$ is the semidiscretization parameter, $A_h : D(A_h) \subset E_h \to E_h$ are closed linear operators in Banach spaces E_h generating C_0-semigroups $\{U_h(t)\}_{t\geq 0}$. We assume that $E_0 = E, A_0 = A, F_0 = F$ and continuous maps $F_h : [0,T] \times E_h \multimap E_h$ satisfying conditions $(F1) - (F4)$ for each $h \in H.$

1. Basic concepts

D e f i n i t i o n 1. A mild solution to the Cauchy problem for inclusion (1) with initial condition

$$x(0) = x_0 \tag{5}$$

on an interval $[0,\tau] \subseteq [0,T]$ is a function $x \in C([0,\tau]; E)$ which can be represented as

$$x(t) = \mathcal{G}(t)x_0 + \int_0^t (t-s)^{q-1}\mathcal{T}(t-s)\phi(s)ds, \qquad t \in [0,T],$$

where $\phi \in \mathcal{P}^{\infty}_F(x)$,

$$\mathcal{G}(t) = \int_0^{\infty} \xi_q(\theta)U(t^q\theta)d\theta, \qquad \mathcal{T}(t) = q\int_0^{\infty} \theta\xi_q(\theta)U(t^q\theta)d\theta,$$

$$\xi_q(\theta) = \frac{1}{q}\theta^{-1-\frac{1}{q}}\Psi_q(\theta^{-1/q}),$$

$$\Psi_q(\theta) = \frac{1}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\theta^{-qn-1}\frac{\Gamma(nq+1)}{n!}\sin(n\pi q), \theta \in \mathbb{R}_+.$$

By the symbol $\Sigma_{x_0}^F[0,\tau]$ we will denote the set of all mild solutions to the Cauchy problem (1), (5) on an interval $[0,\tau] \subseteq [0,T]$. In articles [3] and [4] we proved the following local existence result.

Theorem 1. *Under conditions* (A), $(F1)-(F4)$ *there exists* $\tau \in (0,T]$ *such that* $\Sigma_{x_0}^F[0,\tau]$ *is a nonempty subset of the space* $C([0,\tau];E)$.

D e f i n i t i o n 2. We will say that problem (1), (5) satisfies condition (Q) provided:

$(Q1)$ $\Sigma_{x_0}^F[0,T]$ is a non-empty compact subset of $C([0,T];E)$;

$(Q2)$ the following extendability condition holds:

$$\Sigma_{x_0}^F[0,\tau] = \Sigma_{x_0}^F[0,T]|_{[0,\tau]}$$

for every $\tau \in (0,T]$.

We suppose that there exist linear operators $Q_h : E_h \to E, h \in H, Q_0 = I$ and projection operators $P_h : E \to E_h,\ \ P_0 = I$ such that

$$P_h Q_h = I_h, \tag{6}$$

where I_h is the identity on E_h and

$$Q_h P_h x \to x \tag{7}$$

as $h \to 0$ for each $x \in E$. We suppose that the operators P_h and Q_h are uniformly bounded

$$\|P_h\| \leq 1, \quad \|Q_h\| \leq 1 \tag{8}$$

for all $h \in H$.

An initial condition for equation (4) will be given by the equality

$$x_h(0) = x_h(T) (\text{or } x_h(0) = -x_h(T)) \tag{9}$$

2. Main results

Theorem 2. *Under conditions* $(A),(F1)-(F2),(Q1)-(Q2)$ *let problem* (1) - (2) (*or* (1) - (3)) *has the solution* $x^\star$ *on the interval* $[0,a]$. *Then, for a sufficiently small* $h>0$ *problems* (4), (9) *have solutions* x_h *on the interval* $[0,a]$ *and*

$$Q_h x_h \to x^\star$$

as $h \to 0$.

Various applications of the theory of differential inclusions of fractional order can be found in papers [5], [6] and [7].

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О МЕТОДЕ ПОЛУДИСКРЕТИЗАЦИИ ДЛЯ ДИФФЕРЕНЦИАЛЬНЫХ ВКЛЮЧЕНИЙ ДРОБНОГО ПОРЯДКА

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Аннотация. В докладе приводится схема полудискретизации для полулинейных дифференциальных включений дробного порядка
Ключевые слова: дифференциальное включение дробного порядка; полулинейное дифференциальное включение; задача Коши; аппроксимация; полудискретизация; неподвижная точка; уплотняющее отображение; мера некомпактности

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